The Davis-Skodje Problem

Reference papers: M.J. Davis and R.T. Skodje, *J. Chem. Phys.* **111**, 859 (1999) and S. Singh, J.M. Powers, and S. Paolucci, *J. Chem. Phys.* **117**,1482 (2002).

In order to test methods for generating low-dimensional slow manifolds, it is useful to have a system with an exact slow manifold. The Davis and Skodje model consists of a two-dimensional system which models a spatially homogeneous premixed reactor and is given by

$$\frac{dy_1}{dt} = -y_1, \tag{1}$$

$$\frac{dy_2}{dt} = -\gamma y_2 + \frac{(\gamma - 1)y_1 + \gamma y_1^2}{(1 + y_1)^2}, \tag{2}$$

where $\gamma > 1$ gives a measure of stiffness for the system. If γ is increased, stiffness will increase. The system has one equilibrium at $y_1 = y_2 = 0$, which is stable, and an exact slow invariant manifold, given by

$$y_2 = \frac{y_1}{1 + y_1}, \tag{3}$$

along which all solution trajecrtories approach it. The basic problem is to efficiently and accurately compute the manifold. In addition, characterize the attractiveness of the slow manifold as a function of γ, and given arbitrary initial conditions, provide an efficient strategy to determine approximately the time it takes for the solution trajectory to be sufficiently close to the slow manifold and the corresponding location near the manifold.

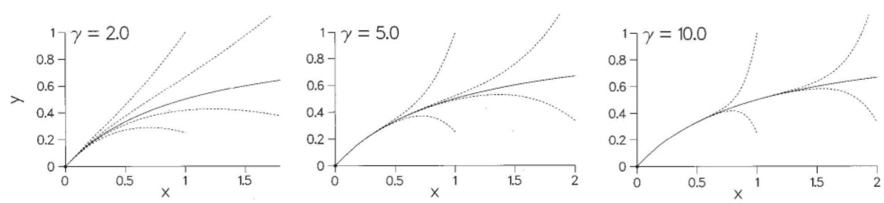

Figure 1: Solutions of (1) and (2) for $\gamma = 2$, 5, and 10. The exact slow manifold (3) is plotted as a solid curve and four representative trajectories are plotted as dashed curves for each value of γ. Trajectories approach the slow manifold more rapidly as γ gets larger.